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# Nonperturbative Inconsistency of Stochastic Quantization in Odd Dimensions

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**Abstract.** It is shown that a stochastically-quantized theory of interactıng fermion and gauge fields in odd spacetime dimensions can be renormalized, preserving both gauge- and parity-invariance. Thus, the pertinent parity-violating anomalies are not reproduced by the stochastic quantization. Moreover, this theory does not possess a nonperturbative equilibrium limit unless one introduces an appropriate parity-violating counterterm. We conclude that an odd-dimensional gauge theory with fermions cannot be inconsistently quantized in the stochastic scheme unless the parity-violating anomales cancel.

## 1. Introduction

The stochastic quantization scheme (SQS) for field-theory models [1] has attracted a lot of interest in the last few years due to some remarkable features in handling gauge-fixing problems in non-Abelian gauge theories [1, 2], models with nonlinear constraints [3], applications to numerical simulations [4] and providing deeper insight into the structure of supersymmetric theories [5]. SQS offers some new invariant regularizations [6] and its consistency was checked when deriving from SQS the correct chiral anomalies in gauge theories with chiral fermions [7–9]★★.

Recently, however, some problems in the application of SQS to models possessing nonperturbative properties due to the nontrivial topology of their configuration spaces were observed [7, 11]. Namely, SQS was shown not to enforce at finite stochastic time quantization of physical parameters in theories with multivalued actions (e.g., SQS average are well defined at finite stochastic time for any values of the coefficient of Chern–Simons terms in odd spacetime dimensions $D$ and of Wess–Zumino terms in even $D$, respectively). On the other hand, for quantized massless fermions interacting with an external background gauge field in odd $D$, SQS fails to reproduce [7] the pertinent parity-violating anomalies (PVA) [12]. (Here, 'parity' means 'space-reflection'.)

However, this last feature of SQS is still not sufficient to answer completely the

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★★ References [10] also discuss chiral anomalies within the SQS. However, these authors' conclusion about the impossibility of describing chiral fermions in SQS for finite stochastic time is incorrect (for a criticism, see the second Ref. [8]).

question about the nonperturbative consistency of SQS in odd $D$. The reason is that the regularized fermion effective action

$$S_F^{\mathrm{eff}}[A] = -\ln\det[-i\nabla(A)]^{(\mathrm{reg})}, \qquad (\nabla(A) = \gamma_\mu \nabla_\mu = \gamma_\mu(\partial_\mu + iA_\mu)), \tag{1}$$

in which PVA appear [see Equations (2) and (3) below], simply cancels out in the equilibrium-limit fermion correlation functions in a background gauge field $A_\mu(x)$. [We shall explicitly consider the case of U($n$) gauge fields $A_\mu(x) = A_\mu^a(x)T^a$, where $\{T^a\}$, $a = 0, 1, \ldots, n^2 - 1$ form a basis of Hermitian U($n$)-generators.] Thus, in the case of background fields $A_\mu(x)$, the uncontradictable coexistence of the following two properties takes place:

(i) Manifest preservation of both gauge- and parity-invariance in the SQS fermion correlation functions through invariant stochastic regularization (e.g., that of the first Ref. [6]) of the fermion propagators in the background field $A_\mu(x)$;
(ii) The appearance of PVA in $S_F^{\mathrm{eff}}[A]$ (Equation (1)) [12] provided (cf. Refs. [13]): $2n > D$ = odd, i.e., the homotopy group $\pi_D(\mathrm{U}(n)) = \mathbb{Z}$, and $N_f$ (number of fermion flavours) = odd simultaneously.

Let us note that $S_F^{\mathrm{eff}}[A]$ does not enter SQS at all, since (1) can never be represented as an SQS average of a certain functional of the fermion fields.

It is the aim of the present Letter to analyze, by means of a nonperturbative method, the full stochastically-quantized theory of both fermion *and* gauge fields in odd $D$ with respect to PVA.

At the end of this introduction, let us recall the form of the regularized $S_F^{\mathrm{eff}}[A]$ (Equation (1)) [12]:

$$\ln\det[-i\nabla(A)]^{(\mathrm{reg})} = \tfrac{1}{2}\ln\det[\nabla^2(A)]^{(\mathrm{reg})} - i\,\frac{\pi}{2}\,\eta_{\nabla(A)} - S_{\mathrm{c.t.}}[A] \tag{2}$$

where

$$\ln\det[\nabla^2(A)]^{(\mathrm{reg})} = -\frac{\mathrm{d}}{\mathrm{d}s}\,\zeta_{\nabla^2(A)}(s)\Big|_{s=0}$$

is regularized by the $\zeta$-function method [14], PVA are contained in $\eta_{\nabla(A)}$ denoting the parity-odd spectral asymmetry-measuring $\eta$-invariant [15] of the Dirac operator $\nabla(A)$ and $S_{\mathrm{c.t.}}[A]$ represents a local gauge-invariant counterterm accounting for the renormalization ambiguity.

Let us recall the identity

$$\eta_{\nabla(A)} = 2W_{\mathrm{ChS}}^{(D)}[A] + 2I^{(D+1)}[A] \tag{3}$$

where $W_{\mathrm{ChS}}^{(D)}[A]$ denotes the well-known Chern–Simons secondary class (e.g., Ref. [16]) and $I^{(D+1)}[A]$ is the index of an appropriate $D + 1(= \text{even})$-dimensional extension of the Dirac operator (cf. the last Ref. [12]). Finally, let us point out that although Equation (2) was originally obtained from spectral decomposition considerations, it may equivalently be derived by means of Pauli–Villars regularization. In the following, indices and arguments will often be suppressed for brevity.

## 2. Functional Integral Formulation of SQS

In this section we derive an expression for the generating functional of gauge-invariant SQS averages for gauge theories with fermions. It takes the form of a generating functional of an effective $D + 1$-dimensional theory. This is a nontrivial extension of the usual superspace functional formulation of SQS [17], since in the present case the fermionic Langevin equations contain a nontrivial kernel $[m - i\nabla(A^{(\eta)})]$ (cf. (5a) and (5b) below] which, in particular, leads to an explicit breaking of the supersymmetry of the effective $D + 1$-dimensional theory.

The models under consideration are described by the following classical action:

$$\begin{aligned} S[A, \psi, \bar{\psi}] &= S_{\mathrm{YM}}[A] + S_F[A, \psi, \bar{\psi}], \\ S_{\mathrm{YM}}[A] &= (4ng^2)^{-1} \int \mathrm{d}^D x \, \mathrm{tr}\,[F^2_{\mu\nu}(A)], \\ S_F[A, \psi, \bar{\psi}] &= - \int \mathrm{d}^D x \bar{\psi}(x)\,[i\nabla(A) + m]\,\psi(x); \\ F_{\mu\nu}(A) &= \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]. \end{aligned} \tag{4}$$

The SQS Langevin equations corresponding to (4) are taken in the following form:

$$\partial_\tau \psi^{(\eta)}(\tau, x) = -[\nabla^2(A^{(\eta)} + m^2]\,\psi^{(\eta)}(\tau, x) + [m - i\nabla(A^{(\eta)})]\,\eta(\tau, x), \tag{5a}$$

$$\partial_\tau \bar{\psi}^{(\eta)}(\tau, x) = -[\nabla^2(A^{(\eta)}) + m^2]^T \bar{\psi}^{(\eta)}(\tau, x) + \bar{\eta}(\tau, x), \tag{5b}$$

$$\begin{aligned} \partial_\tau A^{a(\eta)}_\mu(\tau, x) = {} & -[\nabla_\nu F_{\mu\nu}(A^{(\eta)})]^a(\tau, x) - \\ & - g^2 \bar{\psi}^{(\eta)}(\tau, x) T^a \gamma_\mu \psi^{(\eta)}(\tau, x) + g\eta^a_\mu(\tau, x), \end{aligned} \tag{6}$$

(in perturbation theory in $g$ one should make the rescaling $A_\mu \to gA_\mu$), with initial conditions at $\tau = \tau_0$ (in particular, one can take $\tau_0 \to -\infty$):

$$\psi^{(\eta)}(\tau_0, x) = 0, \qquad \bar{\psi}^{(\eta)}(\tau_0, x) = 0, \qquad A^{(\eta)}_\mu(\tau_0, x) = -iu^{-1}(x)\,\partial_\mu u(x) \tag{7}$$

with $u(x) \in U(n)$, and with the following correlation functions of the Gaussian random sources $\overset{(-)}{\eta}$, $\eta_\mu$:

$$\langle \eta(\tau, x)\bar{\eta}(\tau', x') \rangle = 2\delta(\tau - \tau')\delta^{(D)}(x - x'),$$

$$\langle \eta^a_\mu(\tau, x)\eta^b_\nu(\tau', x') \rangle = 2\delta^{ab}\delta_{\mu\nu}\delta(\tau - \tau')\delta^{(D)}(x - x').$$

To investigate the problem of PVA within SQS it is sufficient to consider SQS averages of gauge-invariant functionals depending only on $A_\mu$ at equal stochastic time $\tau = t$:

$$\langle \mathcal{F}[A_\mu(\cdot,t)]\rangle_\eta = \int \mathcal{D}\eta\mathcal{D}\bar{\eta}\mathcal{D}\eta_\mu \mathcal{F}[A_\mu^{(\eta)}(\cdot,t)] \times \\ \times \exp\left\{-\int \mathrm{d}^D x\, \mathrm{d}\tau[\tfrac{1}{2}\bar{\eta}\eta + \tfrac{1}{4}\eta_\mu^a\eta_\mu^a]\right\} \tag{8}$$

where $A_\mu^{(\eta)}$ denotes the solution to the coupled system (5), (6) with initial conditions (7).

To proceed, we solve explicitly (5a) and (5b) accounting for (7), and insert the solutions into (6):

$$\partial_\tau A_\mu^{a(\eta)}(\tau,x) = -[\nabla_\nu F_{\mu\nu}(A^{(\eta)})]^a(\tau,x) - g^2(\bar{\eta}G_{(-)}[A^{(\eta)}])(\tau,x) \times \\ \times T^a\gamma_\mu(G_{(+)}[A^{(\eta)}](m - i\nabla(A^{(\eta)}))\eta)(\tau,x) + g\eta_\mu^a(\tau,x), \tag{9}$$

where the Green's functions read

$$G_{(\pm)}[A] = [\pm\partial_\tau + m^2 + \nabla^2(A)]^{-1}(\tau,x;\tau',x'). \tag{10}$$

Now, the effective Langevin equation (9) can be used to perform a functional change of variables in (8) from $\eta_\mu$ to $A_\mu \equiv A_\mu^{(\eta)}$★. Introducing an integration over auxiliary Grassmann algebra-valued field variables $C_\mu^a$, $\bar{C}_\mu^a$ into (8) to represent the functional Jacobian det $\|\delta\eta_\mu^a/\delta A_\nu^b\|$ as a functional integral, and another integration over a bosonic auxiliary field $E_\mu^a$ to render the exponent in (8) quadratic in $\eta$, $\bar{\eta}$, one can perform the resulting Gaussian functional integral over $\eta$, $\bar{\eta}$, to obtain the final formula:

$$\langle \mathcal{F}[A_\mu(\cdot,t)]\rangle_\eta = \int \mathcal{D}A_\mu\mathcal{D}C_\mu\mathcal{D}\bar{C}_\mu\mathcal{D}E_\mu \mathcal{F}[A_\mu(\cdot,t)] \times \\ \times \exp\{-\Sigma_{\mathrm{YM}}[A,C,\bar{C},E] - \Sigma_F^{\mathrm{eff}}[A,C,\bar{C},E]\}. \tag{11}$$

Here the following notations are used:

$$\Sigma_{\mathrm{YM}} = \frac{1}{g^2}\int \mathrm{d}\tau\,\mathrm{d}^D x[E_\mu^a E_\mu^a - iE_\mu^a(\partial_\tau A_\mu^a + g^2\delta S_{\mathrm{YM}}/\delta A_\mu^a) - \\ - \bar{C}_\mu^a(\partial_\tau\delta^{ab}\delta_{\mu\nu} + g^2\delta^2 S_{\mathrm{YM}}/\delta A_\mu^a\delta A_\nu^b)C_\nu^b], \tag{12}$$

$$\Sigma_F^{\mathrm{eff}} = -\mathrm{Tr}\ln[1 + 2(Q_0(m^2) + mQ_1(m^2))], \tag{13}$$

where the operators $Q_{0,1}$ are defined in terms of $G_{(\pm)}$ [Equation (10)] as:

$$Q_0 \equiv -iQ_1\nabla(A) + iG_{(-)}CG_{(+)}C, \tag{14a}$$

$$Q_1 \equiv -iG_{(-)}EG_{(+)} - G_{(-)}(\nabla(A)C + C\nabla(A))G_{(-)}CG_{(+)} + \\ + G_{(-)}CG_{(+)}(\nabla(A)C + C\nabla(A))G_{(+)}, \tag{14b}$$

★ From Equation (9), $\eta_\mu$ is, in fact, a functional of the gauge orbit $\{A_\mu^u(\tau,x)\,|\,A_\mu^u(\tau,x) = u^{-1}(x) \times (A_\mu(\tau,x) - i\,\partial_\mu)u(x), u(x)\in \mathrm{U}(n)\}$. This property is accounted for in Equation (11), on the understanding that the functional measure $\mathcal{D}A_\mu$ is a measure over the gauge orbit space, i.e., this measure includes all the necessary gauge-fixing and ghost terms.

and

$$E \equiv E^a_\mu T^a \gamma_\mu , \qquad \overset{(-)}{C} \equiv \overset{(-)}{C}{}^a_\mu T^a \gamma_\mu$$

denote multiplication operators.

Let us note that whereas $\Sigma_{\mathrm{YM}}$ [Equation (12)] may be rewritten in an explicitly supersymmetric form as a function over the Euclidean superspace $\mathbb{R}^{D+1|2}$ with coordinates $z = (x, \tau \mid \theta, \bar{\theta})$★:

$$\begin{aligned} &\Sigma_{\mathrm{YM}} = \tfrac{1}{2} \int \mathrm{d}^{D+1|2} z \, \mathrm{tr}[\bar{\mathscr{D}} \mathscr{A}_\mu, \mathscr{D} \mathscr{A}_\mu] - i \int \mathrm{d}^2\theta \, \mathrm{d}\tau S_{\mathrm{YM}}[\mathscr{A}] , \\ &\mathscr{A}_\mu(z) = A_\mu(\tau, x) + \bar{\theta} C_\mu(\tau, x) + \bar{C}_\mu(\tau, x)\theta + \bar{\theta}\theta E_\mu(\tau, x) , \\ &\mathscr{D} = \partial/\partial\theta , \qquad \bar{\mathscr{D}} = \partial/\partial\bar{\theta} - i\theta \, \partial_\tau , \end{aligned} \tag{15}$$

$\Sigma^{\mathrm{eff}}_F$ [Equation (13)] explicitly breaks this supersymmetry.

## 3. Regularization of the SQS Fermion Effective Action

The starting Langevin equations (5) and (6), as well as (9), are manifestly gauge- and (when $m = 0$) parity-covariant. Parity (space-reflection) transformations are defined in odd $D$ as follows:

$$\begin{aligned} &\psi^{(P)}(\tau, x) = -i\gamma_1 \psi(\tau, x_p) , \qquad \bar{\psi}^{(P)}(\tau, x) = i\bar{\psi}(\tau, x_p)\gamma_1 , \\ &A^{(P)}_\mu(\tau, x) = (A_0, -A_1, A_2, \ldots, A_{D-1})(\tau, x_p) , \qquad x_p \equiv (x^0, -x^1, x^2, \ldots, x^{D-1}) . \end{aligned} \tag{16}$$

$\eta_\mu$, $\overset{(-)}{C}_\mu$, $E_\mu$ have the same parity-transformation properties as $A_\mu$. In particular, under (16) we have $m \to -m$ in (5), (6) and (9).

In the usual quantization of (4), PVA arise from gauge-invariant regularization of $S^{\mathrm{eff}}_F$ [Equations (1) and (2)] under conditions (ii) of Section 1. PVA can be isolated even for *massive* fermions [7, 13] as the $m$-independent parity-odd part of $S^{\mathrm{eff}}_F[A] = -\ln \det [-(i\nabla(A) + m)]$. Likewise, in the SQS averages (11), PVA may possibly arise as an $m$-independent parity-odd contribution to $\Sigma^{\mathrm{eff}}_F$ [Equation (13)] as a result of its gauge-invariant regularization. To this end, the Pauli–Villars regularization is chosen and for simplicity only the case $D = 3$ will be explicitly considered (the generalization to higher odd $D$ being straightforward).

In the ordinary quantization of (4), gauge-invariant Pauli–Villars regularization of fermionic loops may be achieved by adding to the classical action $S[A, \psi, \bar{\psi}]$ (4) an action of heavy mass spinors $\chi, \bar{\chi}$ with opposite (bosonic) statistics:

$$S[A, \psi, \bar{\psi}, \chi, \bar{\chi}] = S[A, \psi, \bar{\psi}] - \int \mathrm{d}^3 x \, \bar{\chi}[M + i\nabla(A)]\chi + \tilde{S}_{\mathrm{c.t.}}[A] \tag{17}$$

where $\tilde{S}_{\mathrm{c.t.}}[A]$ is, as in Equation (2), a local (gauge-invariant) counterterm accounting

★ For general notions of superspaces, see Ref. [18]).

for the renormalization ambiguity (when $M \to \infty$). SQS applied to Equation (17) means that additional Langevin equations for the Pauli–Villars regulator fields arise:

$$\partial_\tau \chi = -[\nabla^2(A) + M^2]\chi + [M - i\nabla(A)]j\,, \tag{18a}$$

$$\partial_\tau \bar{\chi} = -[\nabla^2(A) + M^2]^T \bar{\chi} + \bar{j}\,, \qquad (18b)$$

$$\langle j(\tau, x)\bar{j}(\tau', x')\rangle = 2\delta(\tau - \tau')\delta^{(D)}(x - x') \qquad (18b)$$

$$\partial_\tau \mathrm{A}^a_\mu = -[\nabla_\nu \mathrm{F}_{\mu\nu}]^a - g^2\bar{\psi}\mathrm{T}^a\gamma_\mu\psi - g^2\bar{\chi}\mathrm{T}^a\gamma_\mu\chi - \\ - \delta\tilde{S}_{\text{c.t.}}[A]/\delta A^a_\mu + \eta^a_\mu\,. \tag{19}$$

Equations (18) and (19) lead to the following regularization of $\Sigma_F^{\text{eff}}$ [Equation (13)] in the SQS generating functional (11):

$$\begin{aligned}\Sigma_F^{\text{eff(reg)}}(m) &= \lim_{M\to\infty}\{\Sigma_F^{\text{eff}}(m) - \Sigma_F^{\text{eff}}(M)\} + \Sigma_{\text{c.t.}}\\ &\equiv \Sigma_F^{(\text{normal})} + \Sigma_F^{(\text{PVA})} + \Sigma_{\text{c.t.}}\,,\\ \Sigma_{\text{c.t.}} &= -i\int \mathrm{d}^2\theta\,\mathrm{d}\tau\,\tilde{S}_{\text{c.t.}}[\mathscr{A}]\,,\end{aligned} \tag{20}$$

where the parity-normal and the parity-violating parts of $\Sigma_F^{\text{eff(reg)}}$ are explicitly separated and the counterterm $\Sigma_{\text{c.t.}}$ is written in the superspace notation (15).

In $D = 3$, according to (15), (5) and (6), we have the following field scale dimensions:

$$\dim A_\mu = 1\,, \qquad \dim \overset{(-)}{C}_\mu = 2\,, \qquad \dim E_\mu = 3\,. \tag{21}$$

Then, from dimensional and parity-transformation arguments using (14b), (21) and (16), one can easily deduce that the only PVA contribution to (20) may arise from the first term in the expansion of $\Sigma_F^{\text{eff}}$ [Equation (13)] in powers of $Q_{0,1}$:

$$\Sigma_F^{(\text{PVA})} = \lim_{M\to\infty}\{2M\,\mathrm{Tr}[Q_1(M^2)]\}\,. \tag{22}$$

To compute (22), one can use the 'proper-time' representation for $G_{(\pm)}$ [Equation (10)] entering $Q_1$ [Equation (14b)]:

$$\begin{aligned}&G_{(\pm)}(\tau, x; \tau', x')\\ &= \int_0^\infty \mathrm{d}\alpha[M^2 + \nabla^2(A) \mp \partial_\tau]\exp\{-\alpha[M^2 + \nabla^2(A) \pm \partial_\tau][M^2 + \nabla^2(A) \mp \partial_\tau]\}\,.\end{aligned} \tag{23}$$

After rescaling $\alpha \to \beta = \alpha M^4$ in (23), the behaviour of (23) in the limit $M \to \infty$ is determined by the asymptotic Seeley–De Witt expansion of the corresponding 'heat kernel' for small $\beta$. (The coefficient functions in this expansion may be systematically calculated by means of the symbol calculus of (pseudo) differential operators, cf. for example Ref. [19].) The result of this computation for (22) is:

$$\Sigma_F^{(\mathrm{PVA})} = -\frac{1}{8\pi}\int \mathrm{d}\tau\,\mathrm{d}^3x\,\varepsilon_{\mu\nu\lambda}\,\mathrm{tr}\,[E_\mu F_{\nu\lambda}(A) - 2\bar{C}_\mu \nabla_\nu C_\lambda]$$

$$= -\frac{1}{16\pi}\int \mathrm{d}^2\theta\,\mathrm{d}^3x\,\varepsilon_{\mu\nu\lambda}\,\mathrm{tr}\,[\mathscr{A}_\mu F_{\nu\lambda}(\mathscr{A}) - i\tfrac{2}{3}\mathscr{A}_\mu\mathscr{A}_\nu\mathscr{A}_\lambda]$$

$$= \pi\int \mathrm{d}^2\theta\,\mathrm{d}\tau\,W^{(3)}_{\mathrm{ChS}}[\mathscr{A}]\,, \tag{24}$$

where once again the superspace notation (15) is used.

Although (24) is the $(3 + 1\,|\,2)$-dimensional superspace analogue of the ordinary Chern–Simons term $W^{(3)}_{ChS}[A]$, there is a crucial difference between them. Unlike the ordinary $W^{(3)}_{\mathrm{ChS}}[A]$, (24) *is* gauge-invariant under arbitrary homotopically nontrivial ($\tau$- and $\theta$-independent) gauge transformations. Therefore, we can choose the local gauge-invariant counterterm in (20) in the form:

$$\Sigma_{\mathrm{c\,t}}[\mathscr{A}] = -\pi\int \mathrm{d}^2\theta\,\mathrm{d}\tau\,W^{(3)}_{\mathrm{ChS}}[\mathscr{A}]\,, \tag{25}$$

i.e.,

$$\tilde{S}_{\mathrm{c\,t.}}[A] = -i\pi W^{(3)}_{\mathrm{ChS}}[A] \quad \text{in Equations (17) and (19)}\,, \tag{25$'$}$$

such that the PVA is cancelled completely:

$$\Sigma_F^{\mathrm{eff(reg)}} = \Sigma_F^{(\mathrm{normal})}\,.$$

## 4. Conclusions – Inconsistency of SQS in Odd Dimensions

The main result of Section 3 is that, unlike ordinary quantization in odd $D$, SQS is *free* of PVA for finite stochastic time $t$. In fact, we have shown that SQS averages (11) can be renormalized in such a way that both gauge- and parity-invariance are maintained.

Now, let us analyze what are the implications of the absence of PVA in the renormalized SQS averages (11) for the equilibrium limit:

$$\lim_{t\to\infty}\langle \mathscr{F}[A_\mu^{(\eta)}(\cdot\,,r)]\rangle_\eta$$

$$= \lim_{t\to\infty}\left(\lim_{M\to\infty}\int \mathscr{D}_\mu\mathscr{D}\eta\mathscr{D}\bar{\eta}\mathscr{D}j\mathscr{D}\bar{j}\,\times\right.$$

$$\times\,\mathscr{F}[A_\mu^{(\eta,j)}(\cdot\,,t)]\exp\left\{-\int \mathrm{d}^Dx\,d\tau[\tfrac{1}{4}\eta_\mu^a\eta_\mu^a + \tfrac{1}{2}\bar{\eta}\eta + \tfrac{1}{2}\bar{j}f]\right\}, \tag{26}$$

where $A_\mu^{(\eta,j)}$ is the solution to the regularized effective Langevin equation (19). According to (25′), Equation (19) contains the 'drift' term $i\pi\delta W^{(D)}_{\mathrm{ChS}}/\delta A_\mu^a$. However, as was shown in Ref. [11], SQS averages corresponding to gauge-field Langevin equations with a drift term $i\xi\delta W^{(D)}_{\mathrm{ChS}}/\delta A^a_\mu$ in odd $D$ do *not* possess a nontrivial equilibrium limit [under conditions (ii) of Section 1], unless $\xi$ is quantized as $\xi = 2\pi l, l \in \mathbb{Z}$.

As a consequence, the SQS averages (26) do *not* possess a nontrivial nonperturbative equilibrium limit [under conditions (ii) of Section 1]. On the other hand, the equilibrium limit (26) does exist, provided we set $\tilde{S}_{\mathrm{c.t.}}[A] = 0$ in (17) and (19), i.e., $\Sigma_{\mathrm{c.t.}} = 0$ in (20). This, however, implies parity-violating renormalization of (26) and (11) or, equivalently, this can be interpreted as introducing an additional parity-violating counterterm $S'_{\mathrm{c.t.}}[A] = +i\pi W^{(D)}_{\mathrm{ChS}}[A]$ to the parity-invariant (for $m = 0$) renormalized SQS for gauge fields and fermions in odd $D$.

Finally, we conclude that the conflict between the absence of PVA in SQS and their pertinence to the ordinary quantized theory and, correspondingly, the dependence of the equilibrium limit of the SQS averages (26) and (11) on the renormalization scheme prescriptions should be viewed as the inconsistency of SQS in odd $D$.

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